Modeling Longevity Risk with Generalized Dynamic Factor Models and Vine-Copulas

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- Longevity risk can be understood as a downward deviation of the population mortality rates from a forecasted mortality trend,
- It is a topic of great academic interest in recent times, specifically for the actuarial literature.
- The central point in understanding longevity risk is related to the stochastic nature of mortality rates.
- Hence, the stochastic modeling of mortality rates has been well documented and recommended by the regulators.

- We present a methodology to forecast specific mortality rates and estimate longevity and mortality risks.
- we used Generalized Dynamic Factor Models fitted on the differences of the log-mortality rates. The strategy consists on making the mortality rates dependent on a few numbers of unobserved stochastic factors.
- We compare its forecasting performance with models previously proposed in the literature, such as the traditional Lee-Carter Model.
- We also construct risk measures such as VaR and Tail-VaR by the means of vine-copula simulations.

- Estimation and prediction based on Generalized Dynamic Factor Models.
- Construction of alternative confidence scenarios by the means of vine-copula simulations.

In this way, we provide a robust alternative to measure longevity and mortality risks, using risk measures such as Value at Risk, or Tail-Value at Risk.

Let be N the number of variables. For $i = 1, \ldots, N$, $t = 1, \ldots, T$ a Factor Model can be defined as:

$$x_{it} = \lambda_i^{'} F_t + e_{it}$$

$$x_{it} = C_{it} + e_{it}$$

C_{it} common component
e_{it} idiosyncratic error
λ_i factor loadings
F_t common factors

Estimation by Principal Components and Generalized Pricipal Components.

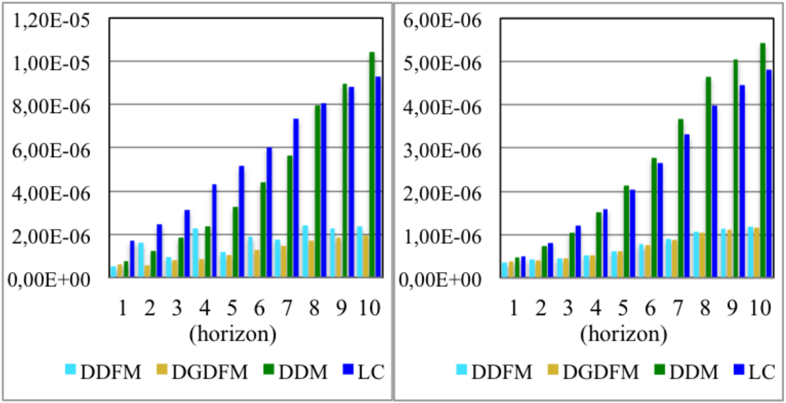
- The dependence arising in the “noise-high” frequency of the spectra is key to the estimation of unexpected movements in the time series.
- We can estimate dependence in a very general way by the means of copulas.
- A copula is a multivariate probability distribution such that:

$$(e_1, \ldots, e_N) = C(F_1(e_1), \ldots, F_N(e_N)),$$

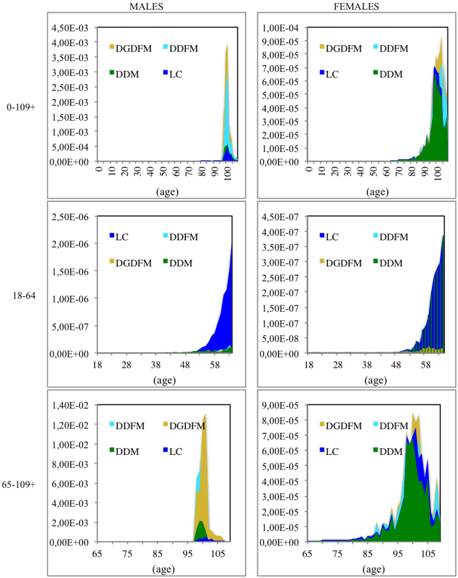
where C is the copula and N the number of mortality rates (We can do this using conditional bivariate copulas)

- The methodology is used to forecast mortality rates and estimate risk measures for the United Kingdom. Data from 1950 to 2011 (Human Mortality Database).
- 1950 to 2001 (we estimate the models) and 2002 to 2011 (to measure their relative performance).
- The Mean Squared Forecasting Error (MSFE) for the changes in log-motility rates for several forecasting horizons is estimated for males and females as a weighted average of the individual forecasting errors, the weights being the population in each category for the cases considered in the exercise

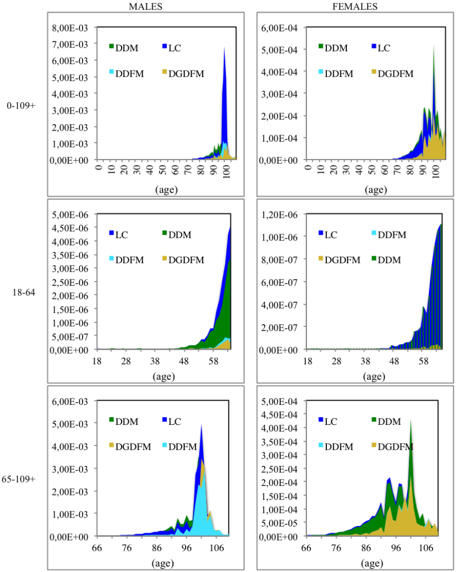
MSFE for population 0-109 years (Males and Females)



MSFE one year ahead



MSFE ten years ahead



LONGEVITY AND MORTALITY RISKS FOR THREE
PORTFOLIOS OF SIZE 30,000 (FORECASTING ONE YEAR AHEAD)

		TVaR	VaR	Expected Deaths	VaR	TVaR
	Level	**0.5**	**0.5**		**99.5**	**99.5**
	0-109+	238	239	252	261	262
Males	18-64	78	78	80	82	82
	65-109+	1283	1288	1346	1394	1399
	0-109+	254	255	259	263	263
Females	18-64	52	52	54	56	56
	65-109+	1242	1244	1257	1269	1270

- Introducing dynamics by the means of adding lags (new factors) generates a better fitting of the models to data, especially when modeling male populations;
- ...although it is the differentiation of the series, which increase considerably the forecasting capability of the factors models.
- The gains in terms of forecasting get bigger as the forecast horizon increases.
- The intuition of this finding arises from the fact that traditional factor models on the log-mortality rates, as the Lee-Carter's model, impose a cointegration relationship among all the series in the system by assumption, which is very unlikely to be observed in the data.

- However the models in differences perform worse than the models in levels, for short forecasting horizons at ages above 95 years.
- There is also an asymmetric relation between longevity and mortality risks, which makes it difficult to try to compensate one risk in one population with the other risk in a different exposed population.